第六章 状态变量分析

△系统分析方法：①输入输出法 ②状态变量法

△一般形式

状态方程

$$\lambda_1'(t) = a_{11}\lambda_1(t) + a_{12}\lambda_2(t) + \dots + a_{1k}\lambda_k(t) + b_{11}e_1(t) + b_{12}e_2(t) + \dots + b_{1m}e_m(t)$$

$$\lambda_2'(t) = \dots$$

$$\lambda_3'(t) = \dots$$

$\lambda_k(t)$：状态变量　$e_k(t)$：输入信号

输出方程

$$r_1(t) = C_{11}\lambda_1(t) + C_{12}\lambda_2(t) + \dots + C_{1k}\lambda_k(t) + d_{11}e_1(t) + d_{12}e_2(t) + \dots + d_{1m}e_m(t)$$

$$r_2(t) = \dots$$

$$r_3(t) = \dots$$

$r_k(t)$：输出信号

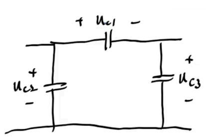
矩阵形式：
$$\left.\begin{aligned} \frac{d\lambda(t)}{dt} &= A\lambda(t) + Be(t) \\ r(t) &= C\lambda(t) + De(t) \end{aligned}\right\} \text{连续}$$

$$\left.\begin{aligned} \lambda(n+1) &= A\lambda(n) + Be(n) \\ r(n) &= C\lambda(n) + De(n) \end{aligned}\right\} \text{离散}$$

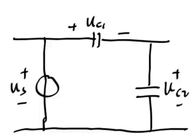
△电路图→状态方程

选独立 $u_C(t)$，$i_L(t)$

独立：无 KCL，KVL 约束关系



任选两个电容电压是独立的



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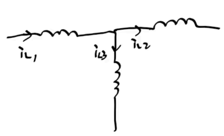
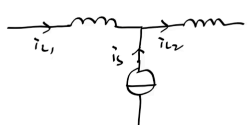
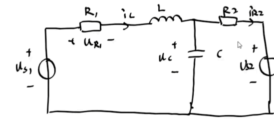


任选两个电感电流是独立



任选一个电感电流是独立的

例：电路如图所示，以电阻 R_1 上的电压 u_{R1} 和电阻 R_2 上的电流 i_{R2} 为输出，列写电路的状态方程和输出方程。



解 状态变量 $i_L(t)$，$u_C(t)$

$$\begin{cases} u_{S1}(t) = \underbrace{u_{R1}(t)}_{= i_L(t)\cdot R_1} + L\cdot\frac{di_L(t)}{dt} + u_C(t) \\ i_L(t) = C\cdot\frac{du_C(t)}{dt} + \underbrace{i_{R2}(t)}_{= \frac{u_C(t)-u_{S2}(t)}{R_2}} \end{cases}$$

$$\therefore \begin{cases} \frac{di_L(t)}{dt} = -\frac{R_1}{L}i_L(t) - \frac{1}{L}u_C(t) + \frac{1}{L}u_{S1}(t) \\ \frac{du_C(t)}{dt} = \frac{1}{C}i_L(t) - \frac{1}{CR_2}u_C(t) + \frac{1}{CR_2}u_{S2}(t) \end{cases}$$

$$\therefore \begin{bmatrix} i_L'(t) \\ u_C'(t) \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{R_2C} \end{bmatrix} \cdot \begin{bmatrix} i_L(t) \\ u_C(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{R_2C} \end{bmatrix} \begin{bmatrix} u_{S1}(t) \\ u_{S2}(t) \end{bmatrix}$$

输出方程　$u_{R1}(t) = i_L(t)\cdot R_1$

$$i_{R2}(t) = \frac{1}{R_2}u_C(t) - \frac{1}{R_2}u_{S2}(t)$$

$$\therefore \begin{bmatrix} u_{R1}(t) \\ i_{R2}(t) \end{bmatrix} = \begin{bmatrix} R_1 & 0 \\ 0 & \frac{1}{R_2} \end{bmatrix} \cdot \begin{bmatrix} i_L(t) \\ u_C(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{R_2} \end{bmatrix} \begin{bmatrix} u_{S1}(t) \\ u_{S2}(t) \end{bmatrix}$$

△信号流图 → 状态/输出方程

(二) 由信号流图建立状态方程和输出方程

方法：① 选择流图中所有的 $\frac{1}{s}$ / $\frac{1}{z}$ 后的点作为状态变量，则在该点之前的点，即为 $\frac{d\lambda(t)}{dt}$ / $\lambda(n+1)$，　($\frac{1}{p}$ / $\frac{1}{z}$)

② 根据流图，将 $\frac{d\lambda(t)}{dt}$ / $\lambda(n+1)$ 和输出表示为状态变量和输入的线性组合。

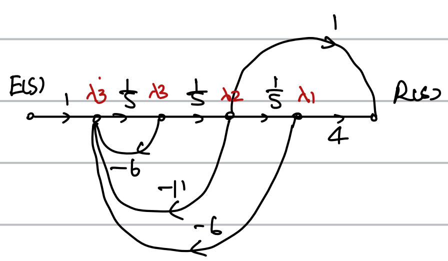
例：试用三种不同结构画出该系统的流图表示，并且列写出对应的状态方程和输出方程。

$$H(s)=\frac{s+4}{s^3+6s^2+11s+6}$$

△解 三种形式：直接、串联、并联

$$H(s)=\frac{\frac{1}{s^2}+\frac{4}{s^3}}{1+\frac{6}{s}+\frac{11}{s^2}+\frac{6}{s^3}}$$

直接：



状态方程

$$\begin{cases}\dot\lambda_3=-6\lambda_3-11\lambda_2-6\lambda_1+e(t)\\ \dot\lambda_2=\lambda_3\\ \dot\lambda_1=\lambda_2\end{cases}$$

$r(t)=\lambda_2+4\lambda_1$ （输出方程）

矩阵：

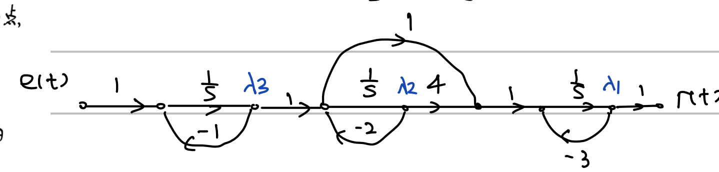
$$\begin{bmatrix}\dot\lambda_1\\ \dot\lambda_2\\ \dot\lambda_3\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\-6&-11&-6\end{bmatrix}\begin{bmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}[e(t)]$$

$$[r(t)]=[4\ \ 1\ \ 0]\begin{bmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{bmatrix}$$

串联形式：$H(s)=\frac{s+4}{s^3+6s^2+11s+6}=\frac{s+4}{(s+2)(s+1)(s+3)}$

$$=\frac{1}{s+1}\cdot\frac{s+4}{s+2}\cdot\frac{1}{s+3}$$

$$=\frac{\frac{1}{s}}{1+\frac{1}{s}}\cdot\frac{1+\frac{4}{s}}{1+\frac{2}{s}}\cdot\frac{\frac{1}{s}}{1+\frac{3}{s}}$$



$$\begin{cases}\dot\lambda_3=-\lambda_3+e(t)\\ \dot\lambda_2=-2\lambda_2+\lambda_3\\ \dot\lambda_1=-3\lambda_1-2\lambda_2+\lambda_3+4\lambda_2=-3\lambda_1+2\lambda_2+\lambda_3\end{cases}$$

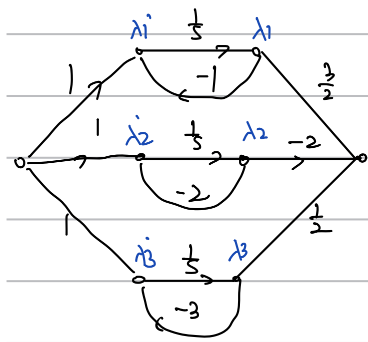
"逆箭头" 注意分支，不能穿过

状态变量，走到状态变量为止

$r(t)=\lambda_1$

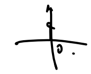
并联：$H(s)=\frac{\frac{3}{2}}{s+1}+\frac{-2}{s+2}+\frac{\frac{1}{2}}{s+3}$

$$=\frac{\frac{3}{2}\cdot\frac{1}{s}}{1+\frac{1}{s}}+\frac{-2\cdot\frac{1}{s}}{1+\frac{2}{s}}+\frac{\frac{1}{2}\cdot\frac{1}{s}}{1+\frac{3}{s}}$$



$$\begin{cases}\dot\lambda_1=e(t)-\lambda_1\\ \dot\lambda_2=e(t)-2\lambda_2\\ \dot\lambda_3=e(t)-3\lambda_3\end{cases}$$

$r(t)=\frac{3}{2}\lambda_1-2\lambda_2+\frac{1}{2}\lambda_3$



△拉氏变换求解状态方程、

$$\begin{cases} \dfrac{d\lambda(t)}{dt} = A\lambda(t) + Be(t) \\ r(t) = C\lambda(t) + De(t) \end{cases}$$

线性. $\mathcal{L}[A\lambda(t) + Be(t)] = A\mathcal{L}[\lambda(t)] + B\mathcal{L}[e(t)]$

两边取拉氏变换

$$\begin{cases} s\lambda(s) - \lambda(0^-) = A\lambda(s) + Be(s) \\ r(s) = C\lambda(s) + de(s) \end{cases}$$

$$(sI - A)\lambda(s) = Be(s) + \lambda(0^-)$$

$$\Rightarrow \lambda(s) = (sI - A)^{-1} \cdot [Be(s) + \lambda(0^-)]$$

$$r(s) = C(sI - A)^{-1}[Be(s) + \lambda(0^-)] + de(s)$$

例

$$\begin{bmatrix} \dot\lambda_1 \\ \dot\lambda_2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e(t)$$

$$r(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} + e(t)$$

$\lambda_1(0^-) = 3$, $\lambda_2(0^-) = 2$, $e(t) = \delta(t)$

$$\begin{bmatrix} s\lambda_1(s) - 3 \\ s\lambda_2(s) - 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} \lambda_1(s) \\ \lambda_2(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} s+1 & -2 \\ 1 & s+4 \end{bmatrix} \begin{bmatrix} \lambda_1(s) \\ \lambda_2(s) \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} \lambda_1(s) \\ \lambda_2(s) \end{bmatrix} = \begin{bmatrix} s+4 & 2 \\ -1 & s+1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 3 \end{bmatrix} \cdot \frac{1}{(s^2+5s+6)}$$

$$= \begin{bmatrix} 3s+18 \\ s \end{bmatrix} \cdot \frac{1}{s^2+5s+6}$$

$$= \begin{bmatrix} \dfrac{3(s+6)}{(s+2)(s+3)} \\ \dfrac{s}{s^2+5s+6} \end{bmatrix} = \begin{bmatrix} \dfrac{12}{s+2} + \dfrac{-9}{s+3} \\ \dfrac{-6}{s+2} + \dfrac{9}{s+3} \end{bmatrix}$$

$$\therefore \begin{bmatrix} \lambda_1(t) \\ \lambda_2(t) \end{bmatrix} = \begin{bmatrix} (12e^{-2t} - 9e^{-3t})\varepsilon(t) \\ (-6e^{-2t} + 9e^{-3t})\varepsilon(t) \end{bmatrix}$$

$r(t)$ 代入即得

△离散.

$$\begin{bmatrix} \lambda_1(n+1) \\ \lambda_2(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix} \begin{bmatrix} \lambda_1(n) \\ \lambda_2(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x(n)$$

$$\begin{bmatrix} y_1(n) \\ y_2(n) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} \lambda_1(n) \\ \lambda_2(n) \end{bmatrix}, \quad \begin{bmatrix} \lambda_1(0) \\ \lambda_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$x(n) = \varepsilon(n)$

解 $$\begin{bmatrix} z\lambda_1(z) - z \\ z\lambda_2(z) - 2z \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix} \begin{bmatrix} \lambda_1(z) \\ \lambda_2(z) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{z}{z-1}$$

$$\begin{bmatrix} z & -1 \\ 6 & z-5 \end{bmatrix} \begin{bmatrix} \lambda_1(z) \\ \lambda_2(z) \end{bmatrix} = \begin{bmatrix} z \\ 2z + \dfrac{z}{z-1} \end{bmatrix}$$

$$\begin{bmatrix} \lambda_1(z) \\ \lambda_2(z) \end{bmatrix} = \begin{bmatrix} z-5 & 1 \\ -6 & z \end{bmatrix} \cdot \frac{1}{z^2-5z+6} \cdot \begin{bmatrix} z \\ 2z + \dfrac{z}{z-1} \end{bmatrix}$$